MA261 Quiz 7

July 15, 2016

Problem 1.

Convert the following integral to spherical coordinates and evaluate.

$$\int_0^2 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{\sqrt{4-z^2-y^2}} \frac{dx\,dy\,dz}{1+(x^2+y^2+z^2)^{\frac{3}{2}}}$$

Solution.

The integral is over a solid ball of radius 2 in the first and second octant.

$$\begin{aligned}
&\int_0^2 \int_0^{\sqrt{4-z^2}} \int_{-\sqrt{4-z^2-y^2}}^{\sqrt{4-z^2-y^2}} \frac{dx\,dy\,dz}{1+(x^2+y^2+z^2)^{\frac{3}{2}}} \\
&= \int_0^{\pi} \int_0^{\pi/2} \int_0^2 \frac{\rho^2 \sin(\phi)\,d\rho\,d\phi\,d\theta}{1+\rho^3} \\
&= \int_0^{\pi} \int_0^{\pi/2} \frac{\ln(9)}{3} \sin(\phi)\,d\phi\,d\theta = \frac{\pi \ln(9)}{3}
\end{aligned}$$

Problem 2.

Set up, but do not evaluate, an integral expression for the volume of a solid bounded by a cone $z^2 = x^2 + y^2$ and a hyperboloid $z^2 = 2 + \frac{x^2+y^2}{2}$ using cylindrical coordinates.

Solution.

$$\int_0^{2\pi} \int_0^2 \int_r^{\sqrt{2+\frac{r^2}{2}}} dz\,r\,dr\,d\theta$$